

How Uncertain Are Solar Neutrino Predictions?

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Abstract

Solar neutrino fluxes and sound speeds are calculated using a systematic reevaluation of nuclear fusion rates. The largest uncertainties are identified and their effects on the solar neutrino fluxes are estimated.

Five solar neutrino experiments (chlorine, Kamiokande, GALLEX, SAGE, and Super-Kamiokande) have measured solar neutrinos with approximately the fluxes and energies predicted by standard solar models, confirming empirically the basic picture of stellar energy generation. However, robust quantitative differences exist between the neutrino experiments and the combined predictions of minimal standard electroweak theory and stellar evolution models. Many authors have suggested that these results provide the first evidence of physics beyond the minimal standard electroweak model.

What are the principal uncertainties in the standard model predictions? In this paper, we determine the uncertainties in the solar neutrino calculations that arise from errors in the nuclear fusion cross sections and show that these uncertainties, while relatively small, are currently the largest sources of recognized errors in the neutrino predictions.

In January, 1997, the Institute for Nuclear Theory (INT) hosted a workshop devoted to determining the best estimates and the uncertainties in the most

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Table 1
Standard Model Predictions (BP98): solar neutrino fluxes and neutrino capture rates, with 1σ uncertainties from all sources (combined quadratically).

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)	Cl (SNU)	Ga (SNU)
pp	$5.94 \left(1.00_{-0.01}^{+0.01}\right)$	0.0	69.6
pep	$1.39 \times 10^{-2} \left(1.00_{-0.01}^{+0.01}\right)$	0.2	2.8
hep	2.10×10^{-7}	0.0	0.0
${}^7\text{Be}$	$4.80 \times 10^{-1} \left(1.00_{-0.09}^{+0.09}\right)$	1.15	34.4
${}^8\text{B}$	$5.15 \times 10^{-4} \left(1.00_{-0.14}^{+0.19}\right)$	5.9	12.4
${}^{13}\text{N}$	$6.05 \times 10^{-2} \left(1.00_{-0.13}^{+0.19}\right)$	0.1	3.7
${}^{15}\text{O}$	$5.32 \times 10^{-2} \left(1.00_{-0.15}^{+0.22}\right)$	0.4	6.0
${}^{17}\text{F}$	$6.33 \times 10^{-4} \left(1.00_{-0.11}^{+0.12}\right)$	0.0	0.1
Total		$7.7_{-1.0}^{+1.2}$	129_{-6}^{+8}

important solar fusion reactions. Thirty-nine experts in low energy nuclear experiments and theory, representing many different research groups and points of view, participated in the workshop and evaluated the existing experimental data and theoretical calculations. Their conclusions have been summarized in a detailed article authored jointly by the participants and to be published by the Reviews of Modern Physics [1]. In general outline, the conclusions of the INT workshop paper confirmed and strengthened previous standard analyses of nuclear fusion rates, although in a few important cases (for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$, ${}^7\text{Be}(p, \gamma){}^8\text{B}$, and ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}$ reactions) the estimated uncertainties were determined to be larger than previously believed.

The purpose of this article is to present calculations of solar neutrino fluxes and solar sound velocities, with special attention to their uncertainties, that were made using the recommended INT nuclear reaction rates and the best available other input data.

Our results can be compared directly with the observed rates in solar neutrino experiments and be used as input for detailed analyses of the particle physics implications of the measured solar neutrino rates. We identify the most important nuclear parameters that need to be measured more accurately in laboratory experiments and determine the precision that is required. By comparing our solar models with five recent, precise helioseismological determinations of

the sound velocities, we estimate the size of the remaining errors in the model calculations.

Table 1 gives the neutrino fluxes and their uncertainties for our best standard solar model (hereafter BP98). The solar model makes use of the INT nuclear reaction rates [1], recent (1996) Livermore OPAL opacities [2], the OPAL equation of state [3], and electron and ion screening as indicated by recent calculations [4]. The adopted uncertainties in input parameters are given in Table 2 and the associated text. We have also made small improvements in our energy generation code, which will be described in detail in a future publication.

The theoretical predictions in Table 1 disagree with the observed neutrino event rates, which are [6]: 2.55 ± 0.25 SNU (chlorine), 73.4 ± 5.7 SNU (GALLEX and SAGE gallium experiments), and $(2.80 \pm 0.19(\text{stat}) \pm 0.33(\text{syst})) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ (^8B flux from Kamiokande).

The principal differences between the results shown in Table 1 and the results presented in our last systematic publication of calculated solar neutrino fluxes [7] is a 1.3σ decrease in the ^8B neutrino flux and 1.1σ decreases in the ^{37}Cl and ^{71}Ga capture rates. These decreases are due principally to the lower $^7\text{Be}(p, \gamma)^8\text{B}$ cross section adopted by Adelberger et al. [1]. If we use, as in our recent previous publications, the Caltech (CIT) value for the ^8B production cross section [8], then the ^8B flux is $\phi(^8\text{B}, \text{CIT}) = 6.1_{-0.9}^{+1.1} \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$, $\Sigma(\phi\sigma)_i \Big|_{\text{Cl, CIT}} = 8.8_{-1.1}^{+1.4}$ SNU, and $\Sigma(\phi\sigma)_i \Big|_{\text{Gallium}} = 131_{-7}^{+9}$ SNU, all of which are within ten percent of the Bahcall-Pinsonneault 1995 best-estimates. The difference between the INT and the CIT estimates of the ^8B production cross section is due almost entirely to the decision by the INT group to base their estimate on only one (the best documented) of the six experiments analyzed by the CIT collaboration.

Table 2 summarizes the uncertainties in the most important solar neutrino fluxes and in the Cl and Ga event rates due to different nuclear fusion reactions (the first four entries), the heavy element to hydrogen mass ratio (Z/X), the radiative opacity, the solar luminosity, the assumed solar age, and the helium and heavy element diffusion coefficients. The $^{14}\text{N} + p$ reaction causes a 0.2% uncertainty in the predicted pp flux and a 0.1 SNU uncertainty in the Cl (Ga) event rates.

The predicted event rates for the chlorine and gallium experiments use recent improved calculations of neutrino absorption cross sections [5]. The uncertainty in the prediction for the gallium rate is dominated by uncertainties in the neutrino absorption cross sections, $+6.7$ SNU (7% of the predicted rate) and -3.8 SNU (3% of the predicted rate). The uncertainties in the chlorine absorption cross sections cause an error, ± 0.2 SNU (3% of the predicted rate),

Table 2

Average uncertainties in neutrino fluxes and event rates due to different input data. The flux uncertainties are expressed in fractions of the total flux and the event rate uncertainties are expressed in SNU. The ${}^7\text{Be}$ electron capture rate causes an uncertainty of $\pm 2\%$ [9] that affects only the ${}^7\text{Be}$ neutrino flux. The average fractional uncertainties for individual parameters are shown. See text for discussion of asymmetric uncertainties and uncertainties due to radiative opacity or diffusion.

<Fractional uncertainty>	pp	${}^3\text{He}{}^3\text{He}$	${}^3\text{He}{}^4\text{He}$	${}^7\text{Be} + p$	Z/X	opac	lum	age	diffuse
	0.017	0.060	0.094	0.106	0.033		0.004	0.004	
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Flux									
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pp	0.002	0.002	0.005	0.000	0.002	0.003	0.003	0.0	0.003
${}^7\text{Be}$	0.0155	0.023	0.080	0.000	0.019	0.028	0.014	0.003	0.018
${}^8\text{B}$	0.040	0.021	0.075	0.105	0.042	0.052	0.028	0.006	0.040
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SNU _s									
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Cl	0.3	0.2	0.5	0.6	0.3	0.4	0.2	0.04	0.3
Ga	1.3	0.9	3.3	1.3	1.6	1.8	1.3	0.20	1.5
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that is relatively small compared to other uncertainties in predicting the rate for this experiment. For non-standard neutrino energy spectra that result from new neutrino physics, the uncertainties in the predictions for currently favored solutions (which reduce the contributions from the least well-determined ${}^8\text{B}$ neutrinos) will in general be less than the values quoted here for standard spectra and must be calculated using the appropriate cross section uncertainty for each neutrino energy [5].

The nuclear fusion uncertainties in Table 2 were taken from Adelberger et al. [1], the neutrino cross section uncertainties from [5], the heavy element uncertainty was taken from helioseismological measurements [10], the luminosity and age uncertainties were adopted from BP95 [7], the 1σ fractional uncertainty in the diffusion rate was taken to be 15% [11], which is supported by helioseismological evidence [12], and the opacity uncertainty was determined by comparing the results of fluxes computed using the older Los Alamos opacities with fluxes computed using the modern Livermore opacities [13]. To include the effects of asymmetric errors, the code `expbrates.f` (see below) was run with different input uncertainties and the results averaged.

Many authors have used the results of solar neutrino experiments and the calculated best-estimates and uncertainties in the standard solar model fluxes to determine the allowed ranges of neutrino parameters in different particle physics models. To systematize the calculations in the solar neutrino predic-

tions, we have constructed an exportable computer code, `exportrates.f`, which evaluates the uncertainties in the predicted neutrino fluxes and capture rates from all the recognized sources of errors in the input data. This code is available at www.sns.ias.edu/~jnb (see Solar Neutrino Software and Data); it contains a description of how each of the uncertainties listed in Table 2 was determined and used.

The low energy cross section of the ${}^7\text{Be} + p$ reaction is the most important quantity that must be determined more accurately in order to decrease the error in the predicted event rates in solar neutrino experiments. The ${}^8\text{B}$ neutrino flux that is measured by the Kamiokande [6], Super-Kamiokande [14], and SNO [15] experiments is, in all standard solar model calculations, directly proportional to the ${}^7\text{Be} + p$ cross section. If the 1σ uncertainty in this cross section can be reduced by a factor of two to 5%, then it will no longer be the limiting uncertainty in predicting the crucial ${}^8\text{B}$ neutrino flux (cf. Table 2).

The ${}^7\text{Be}$ neutrino flux will be measured by BOREXINO [16]. Table 2 shows that the theoretical uncertainty in this flux is dominated by the uncertainty in the measured laboratory rate for the ${}^3\text{He} - {}^4\text{He}$ reaction. In order that the uncertainty from the ${}^3\text{He} - {}^4\text{He}$ cross section be reduced to a level, 3%, that is comparable to the uncertainties in calculating the ${}^7\text{Be}$ flux that arise from other sources, the fractional uncertainty in the ${}^3\text{He} - {}^4\text{He}$ low energy cross section factor must be reduced to 3.5%. This goal is not easy, but it appears to be achievable. The six published determinations of the low energy cross section factor using measurements of the capture γ -rays currently have a 1σ uncertainty of 3.2%, but they differ by 2.5σ (14%) from the value determined by counting the ${}^7\text{Be}$ activity [1].

Could the solar model calculations be wrong by enough to explain the discrepancies between predictions and measurements for solar neutrino experiments? Helioseismology, which confirms predictions of the standard solar model to high precision, suggests that the answer is probably “No.”

Fig. 1 shows the fractional differences between the most accurate available sound speeds measured by helioseismology [17] and sound speeds calculated with our best solar model (with no free parameters). The horizontal line corresponds to the hypothetical case in which the model predictions exactly match the observed values. The rms fractional difference between the calculated and the measured sound speeds is 1.1×10^{-3} for the entire region over which the sound speeds are measured, $0.05R_{\odot} < R < 0.95R_{\odot}$. In the solar core, $0.05R_{\odot} < R < 0.25R_{\odot}$ (in which about 95% of the solar energy and neutrino flux is produced in a standard model), the rms fractional difference between measured and calculated sound speeds is 0.7×10^{-3} .

Helioseismological measurements also determine two other parameters that

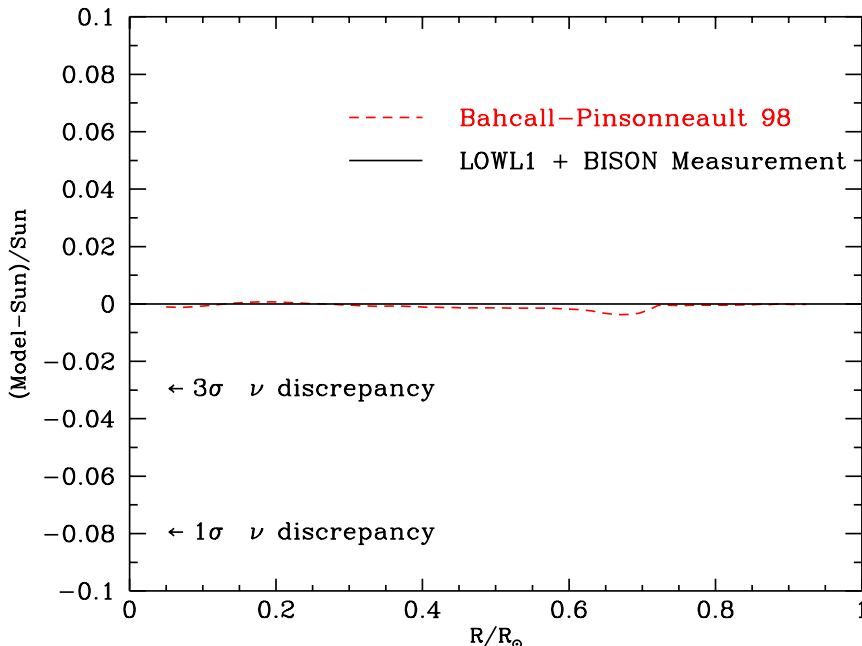


Fig. 1. Predicted versus Measured Sound Speeds. This figure shows the excellent agreement between the calculated (solar model BP98, Model) and the measured (Sun) sound speeds, a fractional difference of 0.001 rms for all speeds measured between $0.05R_{\odot}$ and $0.95R_{\odot}$. The vertical scale is chosen so as to emphasize that the fractional error is much smaller than generic changes in the model, 0.03 to 0.08, that might significantly affect the solar neutrino predictions.

help characterize the outer part of the sun (far from the inner region in which neutrinos are produced): the depth of the solar convective zone (CZ), the region in the outer part of the sun that is fully convective, and the present-day surface abundance by mass of helium (Y_{surf}). The measured values, $R_{\text{CZ}} = (0.713 \pm 0.001)R_{\odot}$ [18], and $Y_{\text{surf}} = 0.249 \pm 0.003$ [10], are in satisfactory agreement with the values predicted by the solar model BP98, namely, $R_{\text{CZ}} = 0.714R_{\odot}$, and $Y_{\text{surf}} = 0.243$. However, we shall see below that precision measurements of the sound speed near the transition between the radiative interior (in which energy is transported by radiation) and the outer convective zone (in which energy is transported by convection) reveal small discrepancies between the model predictions and the observations in this region.

If solar physics were responsible for the solar neutrino problems, how large would one expect the discrepancies to be between solar model predictions and helioseismological observations? The characteristic size of the discrepancies can be estimated using the results of the neutrino experiments and scaling laws for neutrino fluxes and sound speeds.

All recently published solar models predict essentially the same fluxes from the fundamental pp and pep reactions (amounting to 72.4 SNU in gallium experi-

ments, cf. Table 1), which are closely related to the solar luminosity. Comparing the measured and the standard predicted rate for the gallium experiments, the ${}^7\text{Be}$ flux must be reduced by a factor N if the disagreement is not to exceed n standard deviations, where N and n satisfy $72.4 + (34.4)/N = 73.4 + n\sigma$. For a 1σ (3σ) disagreement, $N = 5.1(1.9)$. Sound speeds scale like the square root of the local temperature divided by the mean molecular weight and the ${}^7\text{Be}$ neutrino flux scales approximately as the 10th power of the temperature [19]. Assuming that the temperature changes are dominant, agreement to within 1σ would require fractional changes of order 0.08 in sound speeds (3σ could be reached with 0.03 changes), if all model changes were in the temperature. This argument is conservative because it ignores the contributions from the ${}^8\text{B}$ and CNO neutrinos which contribute to the observed counting rate (cf. Table 1) and which, if included, would require an even larger reduction of the ${}^7\text{Be}$ flux.

We have chosen the vertical scale in Fig. 1 to be appropriate for fractional differences between measured and predicted sound speeds that are of order 0.03 to 0.08 and that might therefore affect solar neutrino calculations. Fig. 1 shows that the characteristic agreement between solar model predictions and helioseismological measurements is more than a factor of 30 better than would be expected if there were a solar model explanation of the solar neutrino problems.

Given the helioseismological measurements, how uncertain are the solar neutrino predictions? We provide a tentative answer to this question by examining more closely the small discrepancies shown in Fig. 1 between the observed and calculated sound speeds.

There has been an explosion of precise helioseismological data in the last year. Fig. 2 compares the results of five different observational determinations of the sound speeds in the sun with the results of our best solar model; the helioseismological discussion in ref. [12] made use of only the LOWL1 data. The small features discrepancies shown in Fig. 2 are robust; they occur when comparisons are made with all the data sets. The vertical scale for Fig. 2 has been expanded by a factor of 20 with respect to the scale of Fig. 1 in order to show the small but robust discrepancies. References to the different helioseismological measurements are given in the caption to Fig. 2.

All five of the precise helioseismological measurements show essentially the same difference between the model and the solar sound speeds near the base of the convective zone. The sharp edge to this feature occurs near the present base of the convective zone at $R_{CZ} = 0.713R_{\odot}$.

What could be the cause of the broad feature shown in Fig. 2 that stretches from about $0.3R_{\odot}$ to about $0.7R_{\odot}$? This feature may be due to some combina-

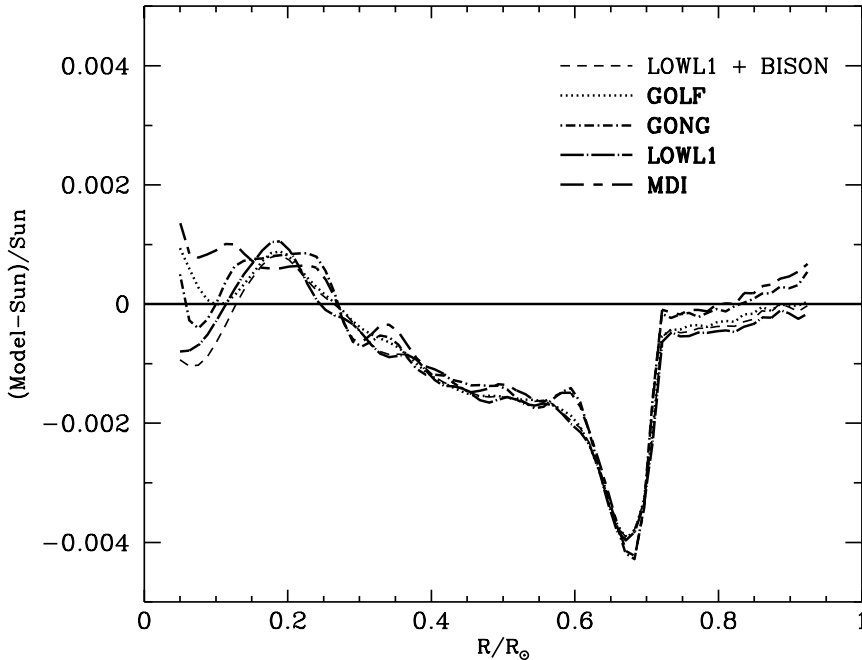


Fig. 2. Five Precise Helioseismological Measurements. The predicted BP98 sound speeds are compared with five different helioseismological measurements [20]. The vertical scale has been expanded by a factor of 20 relative to Fig. 1.

tion of small errors in the adopted radiative opacities or equation of state and the oversimplification we use in our stellar evolution code of a sharp boundary between the radiative and the convective zones. We can make an estimate of the likely effect of hypothetical improved physics on the calculated neutrino fluxes by considering what happens if we change the adopted opacity.

Fig. 3 shows the fractional difference in the computed sound speeds obtained from two solar models that are identical except for the adopted radiative opacity [2], either the version from OPAL92 or the later version from OPAL95. It is apparent from Fig. 3 that the difference in sound speeds caused by using the OPAL95 opacity rather than OPAL92 opacity produces a feature that is similar in shape and in magnitude to the broad feature in Fig. 2. For $0.3 < R/R_{\odot} < 0.6$, the OPAL95 opacity is about 2% less than the OPAL92 opacity for the conditions in the BP98 model. The opacity difference increases to about 6% near the base of the convective zone. A change in opacity of about the same size but opposite in sign to that which occurred between OPAL92 and OPAL95 would remove most of the broad discrepancy, but at the price of producing a slightly deeper convective zone, $0.7105R_{\odot}$, than is observed.

What are the implications for solar neutrino predictions of the existence of the broad 0.2% discrepancy highlighted in Fig. 2? Table 3 shows the neutrino fluxes predicted by the BP98 model constructed using OPAL92 opacities. The

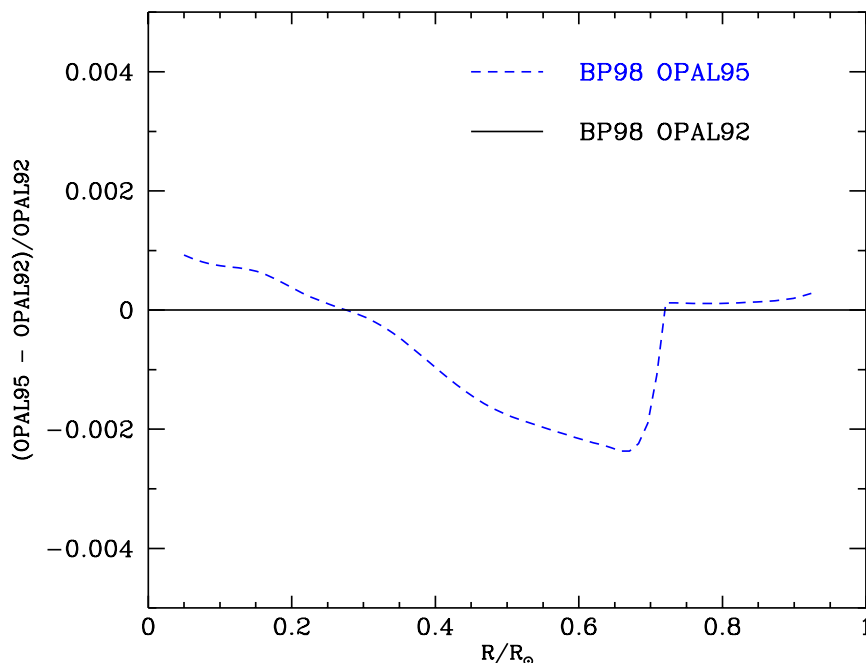


Fig. 3. The effect of opacity on the calculated sound speeds. The figure shows the difference between the calculated sound speeds for two solar models that differ only in the version used of the OPAL opacities [2], 1992 or 1995.

chlorine rate is increased relative to the standard BP98 model (cf. Table 1) by 5% and the ${}^7\text{Be}$ and ${}^8\text{B}$ fluxes are increased by 3% and 5.5% , respectively. These changes are plausible estimates of the changes in the neutrino rates that may be anticipated from further precision improvements in solar models in the outer radiative zone between $0.3R_{\odot}$ and $0.7R_{\odot}$.

The narrow feature near the base of the convective zone may be caused by mixing that is related to the observed [21] depletion of Li and Be. Several independent calculations [22] of the required amount of mixing indicate that the effect on the neutrino fluxes is small, less than a 1% change in the pp flux, and a 2% (4%) decrease in the ${}^7\text{Be}$ (${}^8\text{B}$) neutrino flux.

There is a smaller difference between the models and the observations centered near $R = 0.2R_{\odot}$. This feature occurs using all available helioseismological data and is robust against changes in the method of inversion and inversion parameters. This feature could result, for example, from a 2% inaccuracy in the Livermore opacity at 9×10^6 K [23] or a 0.1% inaccuracy in the OPAL evaluation of the adiabatic index Γ_1 . Detailed calculations [24] of the sensitivity of the neutrino fluxes to changes in opacity or equation of state suggest that either of the changes mentioned above would affect the most sensitive calculated neutrino fluxes by about 2%.

Table 3
Standard solar model with OPAL92 opacities: solar neutrino fluxes. The predicted chlorine capture rate is 8.1 SNU and the gallium rate is 131 SNU.

Source	Flux ($10^{10} \text{ cm}^{-2}\text{s}^{-1}$)
pp	5.92
pep	1.39×10^{-2}
hep	2.08×10^{-7}
${}^7\text{Be}$	4.94×10^{-1}
${}^8\text{B}$	5.44×10^{-4}
${}^{13}\text{N}$	6.25×10^{-2}
${}^{15}\text{O}$	5.52×10^{-2}
${}^{17}\text{F}$	6.59×10^{-4}

In conclusion, we note that three decades of refining the input data and the solar model calculations has led to a predicted standard model event rate for the chlorine experiment, 7.7 SNU, which is very close to the best-estimate value obtained in 1968 [25], which was 7.5 SNU. The situation regarding solar neutrinos is, however, completely different now, thirty years later. Four experiments have confirmed the detection of solar neutrinos. Helioseismological measurements show (cf. Fig. 1) that hypothetical deviations from the standard solar model that seem to be required by simple scaling laws to fit just the gallium solar neutrino results are at least a factor of 30 larger than the rms disagreement between the standard solar model predictions and the helioseismological observations. This conclusion does not make use of the additional evidence which points in the same direction from the chlorine, Kamiokande, and SuperKamiokande experiments. The comparison between observed and calculated helioseismological sound speeds is now so precise ($\sim 0.1\%$ rms) that Fig. 2 indicates the need for an improved physical description of the broad region between $0.3R_{\odot}$ and $0.7R_{\odot}$. The indicated improvement may increase the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes by $\sim 5\%$ (cf. Table 1 and Table 3). The narrow deep feature near $0.7R_{\odot}$ suggests that mixing, possibly associated with Li depletion, might reduce the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes by somewhat less than 5% [22]. Measurements of the low energy cross sections for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction to a 1σ accuracy of 3% and of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction to an accuracy of 5% are required in order that uncertainties in these laboratory experiments not limit the information that can be obtained from solar neutrino experiments about the solar interior and about fundamental neutrino physics.

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